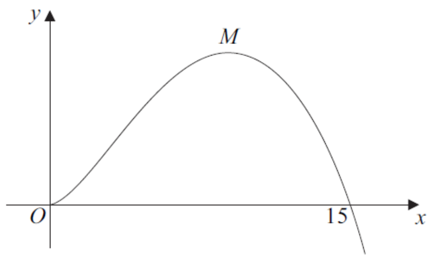
Differentiation

AQA C2 Jun 2009

5 The diagram shows part of a curve with a maximum point M.



The equation of the curve is

$$y = 15x^{\frac{3}{2}} - x^{\frac{5}{2}}$$

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Hence find the coordinates of the maximum point M. *(4 marks)*

(c) The point $P(1, 14)$ lies on the curve. Show that the equation of the tangent to the curve at P is $y = 20x - 6$. *(3 marks)*

(d) The tangents to the curve at the points P and M intersect at the point R. Find the length of RM. *(3 marks)*

Answer: a)

dy/dx = 15 3/2 $x^{1/2}$ – 5/2 $x^{3/2}$

= 45/2 $x^{1/2}$ – 5/2 $x^{3/2}$ = <u>$x^{1/2}$(45-5x)/2</u>

b) At M, dy/dx = 0, so

$x^{1/2}$(45-5x)/2 = 0, so

$x^{1/2}$=0 or 54-5x=0 ⇒x=0 or x=9

From the diagram, at M x≠0, so x=9 and y = 15.$9^{3/2}$-$9^{5/2}$ =

=15.9.3-81.3 = 405-243=162

So M is <u>(9,162)</u>

c) Gradient at P = (45-5)/2=20

So equation of tangent at P is

(y-14)=20(x-1)

y=20x-6

d) Tangent at M is y= 162

This intersects with tangent at P when

162=20x-6

168=20x

x=8.4

so R is (8.4,162)

$RM^2=(9-8.4)^2+(0)$

so RM=0.6

AQA C2 June 2008

1 (a) Write $\sqrt{x^3}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) A curve, defined for $x \geqslant 0$, has equation

$$y = x^2 - \sqrt{x^3}$$

(i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Find the equation of the tangent to the curve at the point where $x = 4$, giving your answer in the form $y = mx + c$. *(5 marks)*

Answer:a) $x^{3/2}$

b) i) $dy/dx = 2x - 3/2x^{1/2}$

ii) At x=4, y=16-4.2 = 8

Slope of tangent at this point = 2.4 – 3/2.2=8-3=5

So equation of tangent at x=4 is

(y-8)=5(x-4)

y=5x-12

(In fact there is an issue with this question, in that the given equation fails the ‘vertical line test’. For any x value, there are two possible y values, depending on whether we take the positive or negative root. At x=4, for example, there is another y value, at 16+8 = 24)

Edexcel C2 June 2005

1. Find the coordinates of the stationary point on the curve with equation $y = 2x^2 – 12x$.

(4)

Answer:

dy/dx=4x-12

At a stationary point, dy/dx=0

so 4x-12=0 ⇒ x=3

so $y=2x^2 – 12x$ = 18-36=-18

Stationary point is (3,-18)

Edexcel C2 Jan 2006

7. The curve C has equation

$y = 2x^3 – 5x^2 – 4x + 2$.

(a) Find $\frac{dy}{dx}$.

(b) Using the result from part (a), find the coordinates of the turning points of C.

(c) Find $\frac{d^2y}{dx^2}$.

(d) Hence, or otherwise, determine the nature of the turning points of C.

Answer:

a) dy/dx = $6x^2$-10x-4

b) At turning points, dy/dx=0

so $6x^2$-10x-4 ⇒ $3x^2$-5x-2=0

(3x+1)(x-2)=0

so x=-1/3 or x=2

If x=-1/3, y = -2/27-5/9+4/3+2 = (-2-15+36+54)/27 = 73/27

If x=2, y = 16-5.4-8+2 = -4-8+2 =-10

So turning points are at (-1/3, 73/27) and (2,-10)

c) $d^2y/dx^2 = 12x-10$

d) At $x=-1/3$, $d^2y/dx^2 =-4-10 = -14$

So x=1/3 is a maximum

At $x=2$, $d^2y/dx^2 = 24-10 = 2$

So x=2 is a minimum