Derivatives of x^n

We have seen that the derivative of x^2 is 2x. We could try to work out x^3, x^4, x^5 and so on. But if we can find the derivative of x^n, where n is an integer, we've done them all.

This time our two points for our generic tangent are (x,x^n) and $(x+h, (x+h)^n)$. So dx=h and

$$dy = (x+h)^n - x^n$$

To multiply out $(x+h)^n$, we need the binomial theorem. We will only show the first three terms. The … reminds us there are more:

$$dy = x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \ \ldots - x^n$$

$$= nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \ \ldots$$

So

$$\frac{dy}{dx} = \frac{nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \ \ldots}{h}$$

$$= nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \ \ldots$$

Our generic tangent is best for very small h, so it is good enough to ignore the second term, which contains h. The terms in the … have h^2, h^3 and so on, and these are even smaller. So

the derivative of x^n is nx^{n-1}.

You must learn this.

Examples

The derivative of x^3 is $3x^2$

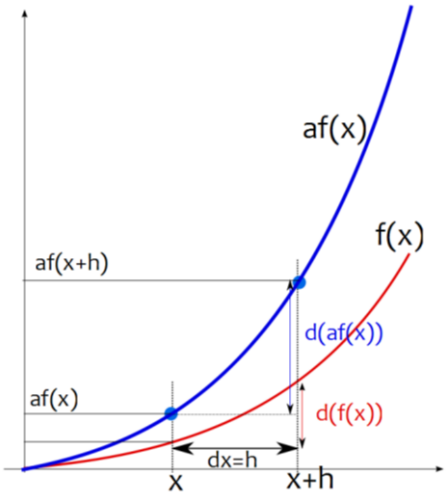
The derivative of x^4 is $4x^3$

The derivative of x^5 is $5x^4$

the derivative of x^n is nx^{n-1}.

You must learn this.

The derivative of af(x)



Suppose we know the derivative of f(x). Can we find the derivative of af(x), where a is a constant? For example, what is the derivative of $3x^2$?

We need to find

$$\frac{d(af(x))}{dx}$$

since this is the slope function applied to the function af(x). Now

$$d\big(af(x)\big) = af(x+h) - af(x)$$

$$= a(f(x+h) - f(x))$$

$$= ad(f(x))$$

So

$$\frac{d(af(x))}{dx} = \frac{ad(f(x))}{dx} = af'(x)$$

Examples

If $f(x)=3x^2$, we can write this as 3g(x), where $g(x)=x^2$. Now $g'(x)=2x$, so

$f'(x) = 3.2x = 6x$

In other words, just differentiate the function without the constant, and stick the constant back on the front.

More examples

function f(x)	derivative f'(x)
$4x^3$	$12x^2$
$9x^5$	$54x^4$
$x^2/2$	x

Derivative of ax^n

Using the above, the derivative of ax^n is anx^{n-1}

Learn this

Derivative of x

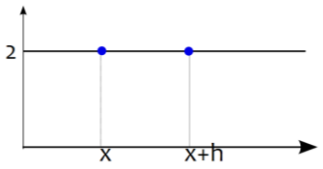
Using the above formula differentiating x gives us x^0. Anything (apart from zero) to the power of zero is 1, so the derivative of x is 1.

Exercise

What is the derivative of 5x?

Derivative of a constant

What is the derivative of, say, 2? Obviously the change in 2 from x to x+h is zero, so the derivative of a constant is zero



Derivative of sums of functions

Suppose we can differentiate two functions, f(x) and g(x). Can we differentiate their sum, f(x)+g(x)?

We need:

$$\frac{d(f(x)+g(x))}{dx} = \frac{f(x+h)+g(x+h)-f(x)-g(x)}{h}$$

$$= \frac{f(x+h)-f(x)+g(x+h)-g(x)}{h}$$

$$= \frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}$$

$$= \frac{d(f(x))}{dx} + \frac{d(g(x))}{dx}$$

$$= f'(x) + g'(x)$$

So we can just differentiate the two functions separately, and add the results.

For example, if $f(x)=x^2$ and $g(x)=x^3$, $u(x)=f(x)+g(x)$, then

$u'(x)=f'(x)+g'(x)=2x+3x^2$

Another example – what is the derivative of f(x)=2x(x-4)?

$f(x)=2x^2-8x$

so $f'(x)=4x-8$

What is the derivative of $4x^2+7$? Answer is 8x

Exercise

Differentiate the following:

x^4

x^{100}

$3x^2$

$7x^2+3x^3$

$(x+2)(x+3)$

$8x^2-7x$

$x-3$