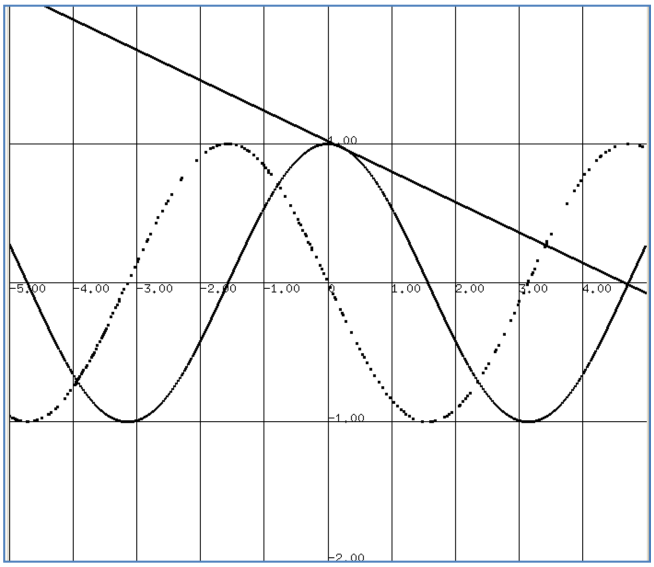
Derivatives of sine and cosine

We can find the derivatives of these algebraically, and review them graphically.

Derivative of cos x



The slope function of cos x looks a bit like sin x, but upside down.

We can work it out algebraically, using the identity for cos(A+B). The slope of our generic tangent is:

$$\frac{\cos(x+h)-\cos(x)}{h}$$

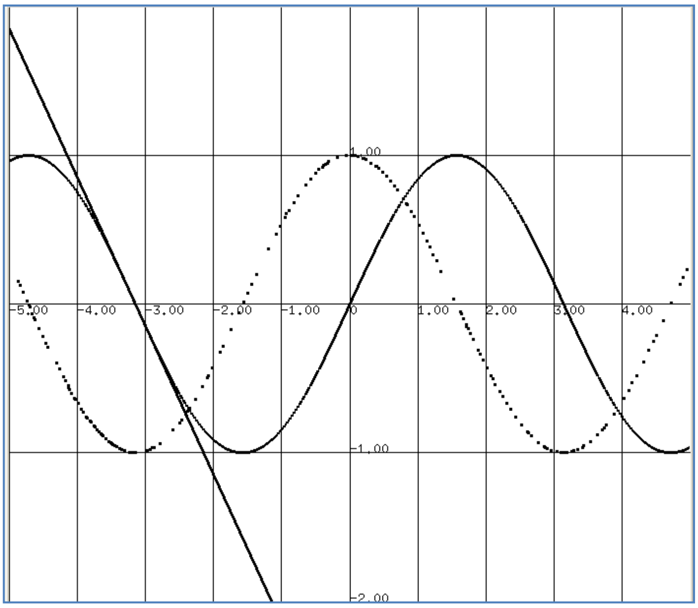
$$=\frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h}$$

Our generic tangents are best when h is small. In that case, cos(h) ≈1 and sin(h) ≈h. So

$$f'(x)=\frac{\cos(x)-\mathrm{h}\sin(x)-\cos(x)}{h}$$

$$=\frac{-\sin(x)\,\mathrm{h}}{h}=-\sin(x)$$

Derivative of sin(x)



This looks a lot like cos(x):

$$\frac{\sin(x+h)-\sin(x)}{h}$$

$$= \frac{\sin(x)\cos(h) + \cos\,(\mathrm{x})\sin\,(h) - \sin\,(x)}{h}$$

Our generic tangents are best when h is small. In that case, cos(h) ≈1 and sin(h) ≈h. So

$$f'(x) = \frac{\sin(x) + \mathrm{hcos(x)} - \sin\,(x)}{h}$$

$$= \frac{\mathrm{hcos(x)}}{h} = \cos\,(x)$$

Exercise One

Differentiate these functions

sin(x)+3x

2x-cos(x)-1

5cos(x)+3x^2

$\frac{\cos(x)+\sin\,(x)}{4}$