Slope of a tangent

How steep is the tangent?

Figure 1 shows a graph of sin(x), from x=-1 to about x=3. Tangents have been drawn at four points. The curve is steepest at x=0, and as x increases it becomes less steep, becoming horizontal at x=π/2 (90 degrees)

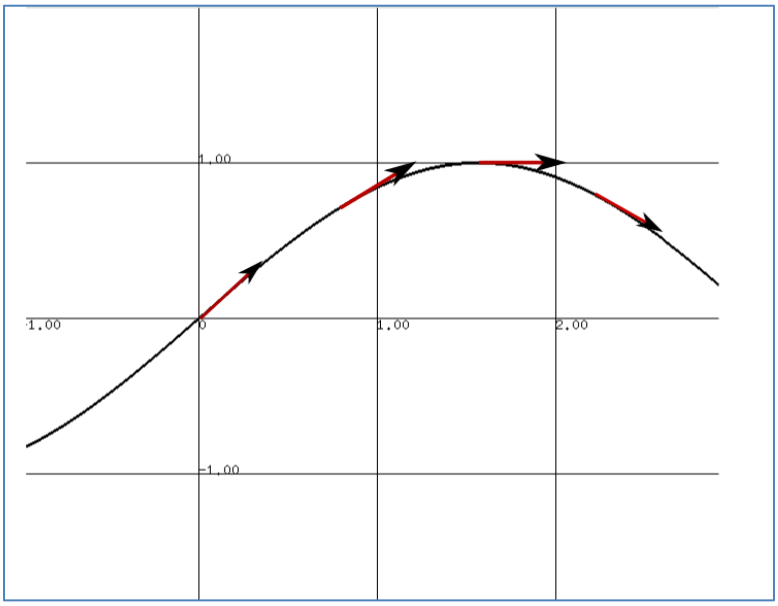


Figure 1

Exercise 1

Where is the tangent to x^2 horizontal?

Slope

Tangents are more or less steep. We put a number to the steepness, to measure the slope of a curve, as shown in Figure 2:

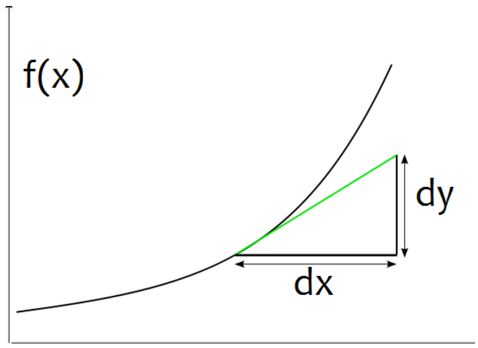


Figure 2

We then define the slope to be

$$slope = \frac{dy}{dx}$$

We can choose dx as any value we like, since dy will be proportionately resized, and the ratio dy/dx will not change.

Exercise 1

Figure 3 shows the graph of x^2, with the tangent drawn at x=1. Measure the slope at x=1.

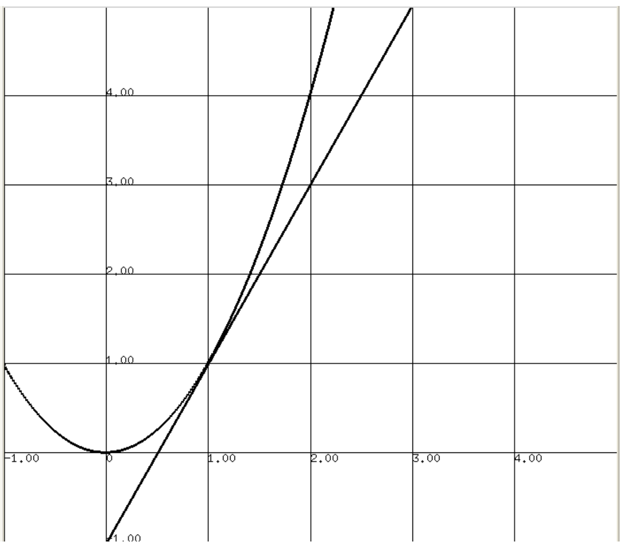


Figure 3

Changing slopes

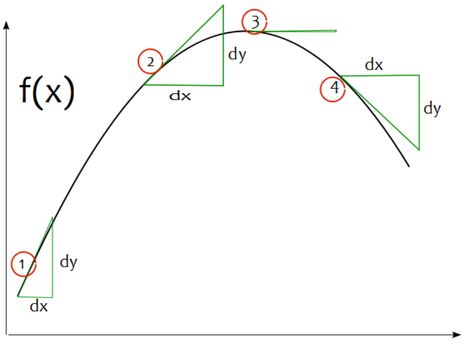


Figure 4

Figure 4 shows the tangent at 4 places on some function of x. At point 1 the function is increasing rapidly. dy is large and so would be the slope dy/dx.

At point 2 the function is increasing less quickly, and so the slope dy/dx would be smaller than at point 1.

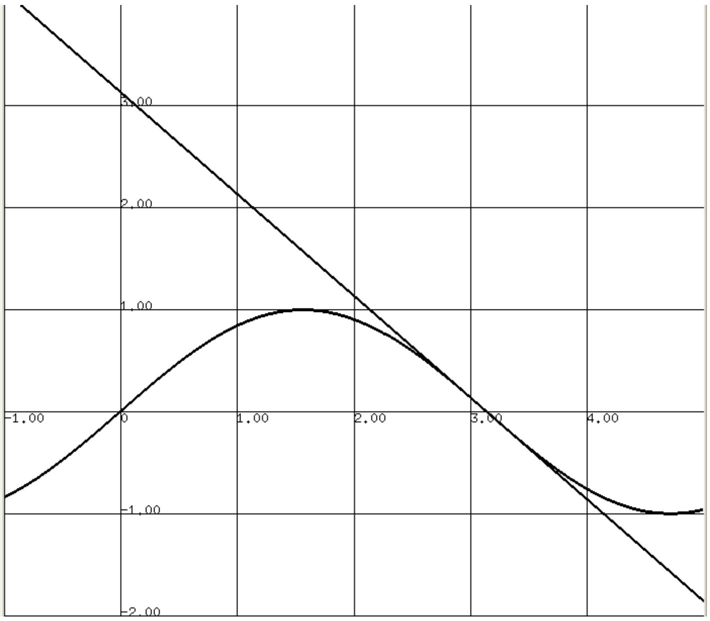
At point 3 the function reaches a maximum. The tangent is horizontal, so dy would be zero, and so would the slope. The slope at a maximum is zero. (Where else would it be zero?)

At point 4 the function is decreasing, and dy is *negative*. This means the slope dy/dx would also be negative. A decreasing function has a negative slope.

Exercise 2

Figure 5 shows the tangent to sin x at x=π radians (180 degrees):

Measure the slope at that point accurately as you can.



sin

as

Figure 5