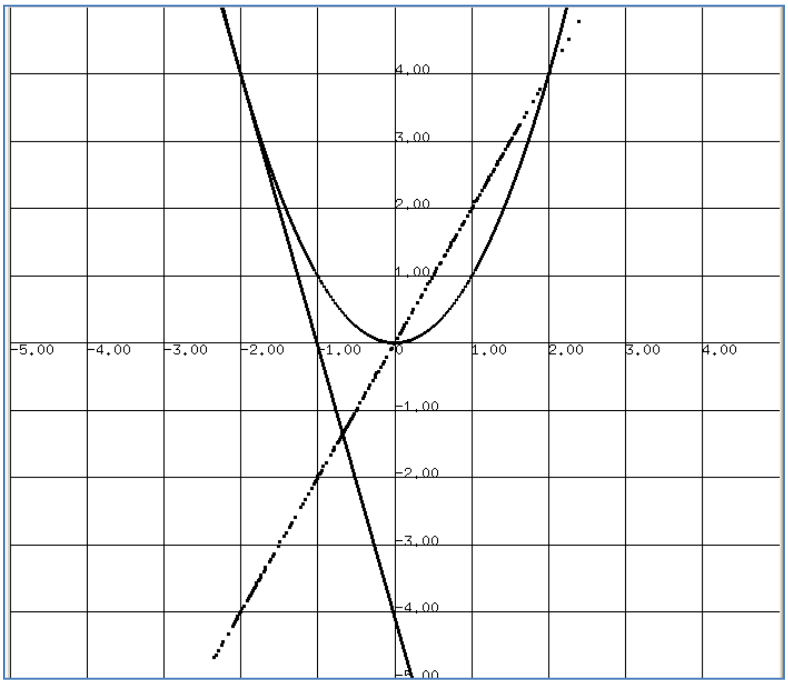
The slope function

We have seen that where a function is locally straight, there is no problem about drawing a generic tangent to it, and we can give a value to the changing slope of the tangent by calculating dy/dx.

Suppose we did that (found dy/dx) at a whole set of points along the function. And if we plotted those values on the graph? This would involve a lot of work, but it is easy on a computer.

On the right we see the function x^2. The straight line line is the tangent at one point. The dots show the slope of the tangent at different x values.



Exercise One

1) For negative values of x, the dots are below the x axis. Why?

2) At x=0, the dot looks like it is zero as well. Why?

3) As x increases beyond x=0, the dots get higher and higher. Why?

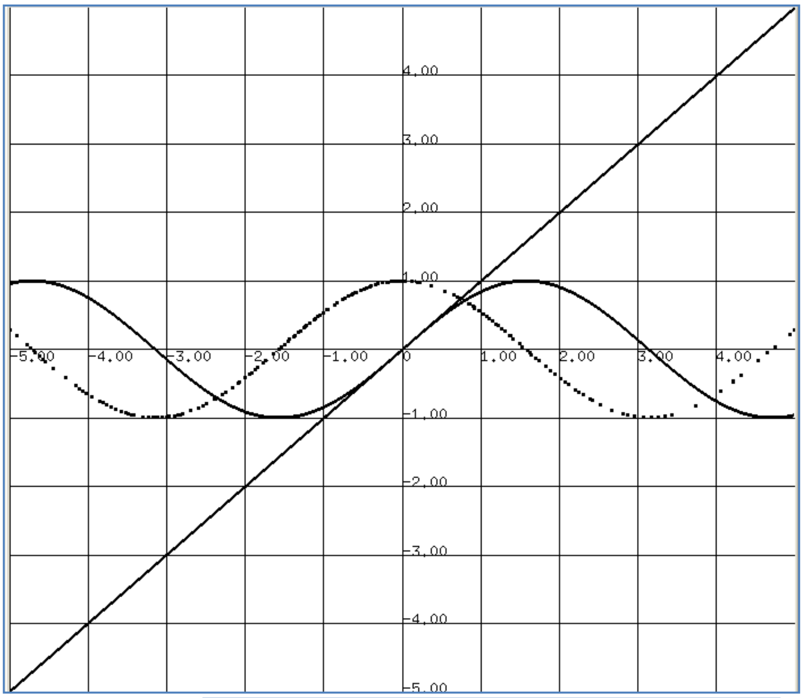
4) Imagine the dots marking out a function. What would be the equation of that line?

The slope function

This means we can think of the changing value of the slope, calculated as dy/dx, as being itself a function. The slope function is called the derivative. If the original function is f(x), the derivative is written as f'(x).

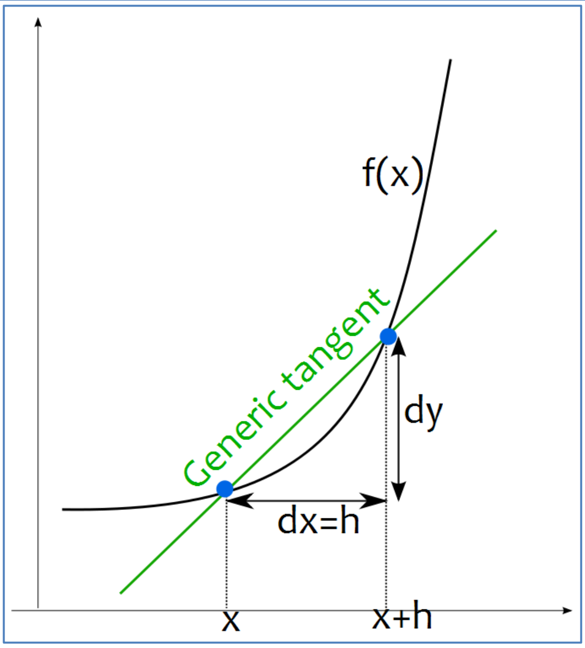
Another example

This shows the slope function, or derivative (as dots), of sin(x) . The generic tangent at x=0 is shown as a straight line. At x=0, the slope function has a value of 1. Why?



Deriving the slope function

Guessing a function on the basis of the shape of its graph is not very reliable. Can we derive it algebraically? To find the slope function, we need to express the slope as a function of x.



We will try to do this for $f(x)=x^2$. We are drawing the generic tangent through two points, (x,x^2) and $(x+h, (x+h)^2)$.

So dx=h, and

$dy=(x+h)^2-x^2$

$=x^2+2hx+h^2-x^2$

$=2hx+h^2$

So our slope function is

$$\frac{dy}{dx} = \frac{2hx + h^2}{h} = 2x + h$$

Our generic tangent is best when h is very small, so our best measure of the slope function or derivative of x^2 is 2x

Notation and differentiation

If we have a function f(x), finding its derivative is often called differentiating the function. This is sometimes written as f'(x), or as

$$\frac{d}{dx} f(x)$$