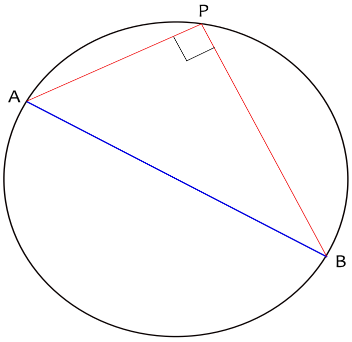
# Circles

## Geometrical properties

### Angle in a semi-circle

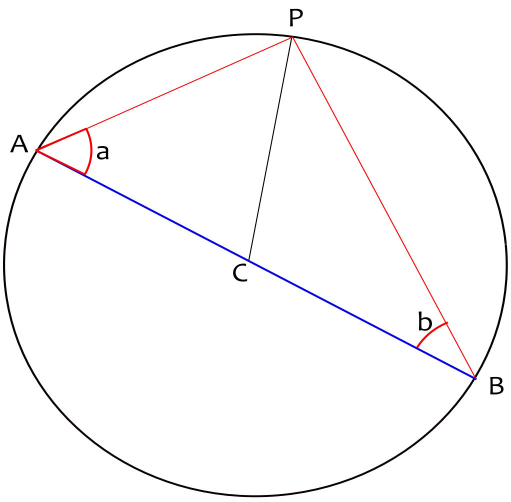
If AB is a diameter, angle APB is a right angle.

This is true no matter where P is on the circle.




Why?

Suppose C is the centre of the circle. Then AC = CP (the radius of the circle) triangle ACP is isoceles, and angle APC = angle PAC = a.




Similarly PCB is isoceles and angle CPB = b.

So the angle APB is a+b

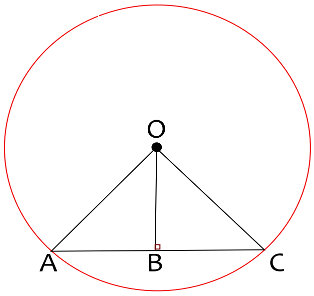
Add up the angles in the triangle ABP – we have a + b + (a+b), and these add up to 180°

So 2a+2b = 180°

So a+b = angle APB = 90°

### Perpendicular to a chord bisects it

Given a chord AC, the line from the centre perpendicular to it bisects it.
In other words, AB = BC

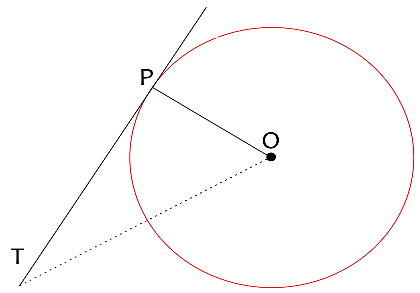



Why? Triangles OCB and OAB are both right-angles triangles. Lengths OC and OB are equal, and OB is common to both.

So if we used Pythagoras to calculate AB abd BC in these two triangles, we will get the same answer. So AB=BC.

### Tangent is perpendicuar to the radius

TP is the tangent to the circle, with P the point of contact and O the centre. Then OPT is a right angle.




Why? A tangent is defined to be a line that touches a circle only once. This means point T must be outside the circle, and distance TO is greater than distance PO. This is true for any point T – so PO is the shortest distance from O to the line.

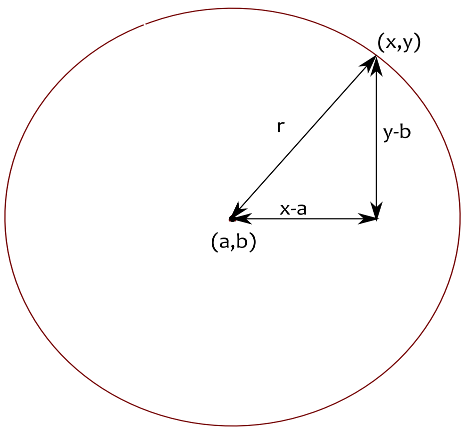
But the shortest distance from a point to a line is the perpendicular. So OPT is a right angle.

## Go-ordinate geometry of a circle

### Equation of a circle

The equation of a circle with centre at (a,b) and radius r is

$(x-a)^2 + (y-b)^2 = r^2$




This is just Pythagoras – see diagram.

**Alternate form** – if we multiply out the above equation:

$x^2-2xa+a^2+y^2-2yb+b^2=r^2$

so

$x^2+y^2-2xa -2yb+a^2+b^2 - r^2 = 0$

or

$x^2+y^2+2gx +2fy+c = 0$

where the circle has centre (-g,-f) and $r = \sqrt{(g^2+f^2-c)}$

## Worked examples

*1. Find the equation of a circle with centre (-3,8) and radius 5*

Answer:

$(x- -3)^2+(y-8)^2=5^2$

or $(x+3)^2+(y-8)^2=25$

*2. A circle has equation*

*$x^2+y^2-12x +8y +43=0$*

*Find its centre and radius*

Answer:

Compare this with

$x^2+y^2+2gx+2fy+c = 0$

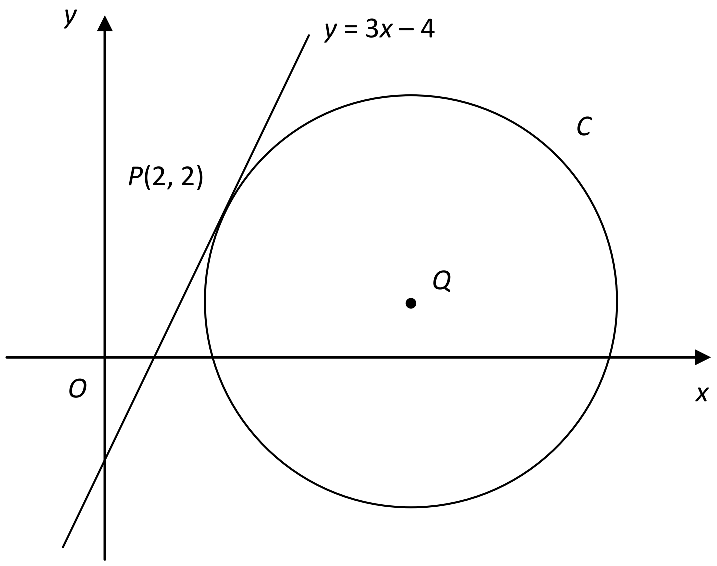
So centre is (6,-4)

and

radius = $\sqrt{(6^2+4^2-43)}$

$=\sqrt{(36+16-43)} = \sqrt{(52-43)}=\sqrt{9} = 3$

*3. (Edexcel C2 May 2006)*




*The line y = 3x – 4 is a tangent to the circle C, touching C at the point P(2, 2), as shown .*

*The point Q is the centre of C.*

*(a) Find an equation of the straight line through P and Q.*

*Given that Q lies on the line y = 1,*

*(b) show that the x-coordinate of Q is 5,*

*(c) find an equation for C.*

Answer

a) The tangent has gradient 3. So the normal has gradient -1/3. The straight line through (2,2) with gradient -1/3 is

$y-2 = -1/3\ (x-2)$

$3y-6=2-x$

$3y+x=8$

b) The point on this line at $y=1$ is

$3+x=8$

So $x=5$

c) Q is (5,1). So the distance $PQ = \sqrt{(3^2+1^2)} = \sqrt{10}$ = radius of circle.

So equation of circle is

$(x-5)^2+(y-1)^2=10$