Standard Integrals

If we know how to diffentiate a function, then we know how to integrate the result. For example,

$$\frac{d}{dx}(\sin x) = \cos x$$

$$so \int \cos x \, dx = \sin x + C$$

The technique is therefore to recognise a function as what you get when you differentiate something – so the integral is that something. For example,

$$\frac{d}{dx}e^x = e^x$$

$$so \int e^x \, dx = e^x + c$$

Sometimes this has to be modified – for example

$$\frac{d}{dx}e^{3x} = 3e^{3x}$$

$$so \int e^{3x} \, dx = \frac{e^{3x}}{3} + c$$

The thought process when seeing $\int e^{3x}$ might be:

1. When you differentiate e^x you get e^x – so the answer to this might be e^{3x}

2. Check by differentiating e^{3x} - you get $3e^{3x}$

3. So we must have $e^{3x}/3$ to get e^{3x}

With practise this can be done quickly.

Example 1

$$\int \sin(4x) \, dx = -\frac{\cos(4x)}{4} + C$$

(If we differentiated cos(4x), we would get -4sin(4x))

Example 2

$$\int \cos(2x+1) \, dx = \frac{\sin(2x+1)}{2} + C$$

(The derivative of sin(2x+1) is 2cos(2x+1))

Table of standard integrals

These are just the derivatives of standard functions ‘backwards’

Integral of..	is..
cos	sin
sin	-cos
sec^2	tan
e^x	e^x
sec tan	sec
cosec cot	-cosec
$cosec^2$	-cot
$1/(1+ x^2)$	$tan^{-1} x$
$1/\sqrt{(1- x^2)}$	$sin^{-1}x$

Example

$$\int 4cosec^2\,(3x+2)dx =$$

$$-\frac{4}{3}\cot(3x+2)+C$$