Integration of x^n

We can integrate x^n easily, by working backwards from what function we would need to have such that when we differentiate it we get x^n.

For example

$$\int x^4\,dx = \frac{x^5}{5} + C$$

since if we differentiate $x^5/5$, we get x^4

In general,

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

or, increase the power by 1 and divide by the new power

Example

$$\int x^3\,dx = \frac{x^4}{4} + C$$

Extensions

We have seen that

$$\int af(x)dx = a\int f(x)dx$$

where a is a constant, and that

$$\int \big(f(x) + g(x)\big)dx = \int f(x)dx + \int g(x)dx$$

We can use these to integrate polynomials easily – for example

$$\int (2x^2 + 4x^{3)}dx =$$

$$2\int x^2\,dx + 4\int x^3 dx =$$

$$2\frac{x^3}{3} + 4\frac{x^4}{4} =$$

$$x^4 + \frac{2x^3}{3}$$

Not 1/x

If you try this on x^{-1}, you would have to raise the power to 0, and divide by this new power – but division by zero is undefined – so this will not work with 1/x.

In fact the integral of 1/x is ln(x) – we'll see why later.